Offer #2024-07170

PhD Position F/M Theoretical and numerical study of dark solitons in nonlinear Schrödinger equations (M/F)

Contract type : Fixed-term contract

Level of qualifications required : Graduate degree or equivalent

Fonction : PhD Position

Level of experience : Recently graduated

About the research centre or Inria department

The Inria University of Lille centre, created in 2008, employs 360 people including 305 scientists in 15 research teams. Recognised for its strong involvement in the socio-economic development of the Hauts-De-France region, the Inria University of Lille centre pursues a close relationship with large companies and SMEs. By promoting synergies between researchers and industrialists, Inria participates in the transfer of skills and expertise in digital technologies and provides access to the best European and international research for the benefit of innovation and companies, particularly in the region.

For more than 10 years, the Inria University of Lille centre has been located at the heart of Lille's university and scientific ecosystem, as well as at the heart of Frenchtech, with a technology showroom based on Avenue de Bretagne in Lille, on the EuraTechnologies site of economic excellence dedicated to information and communication technologies (ICT).

Context

The nonlinear Schrödinger (NLS) equation is ubiquitous in Physics, and appears for example in cold atoms (Bose-Einstein condensates) and nonlinear optics. It is well known that classical versions of this equation have particular solutions, named "solitons", that travel at finite speed, without changing shape over time. These solitons play a central role in the global dynamics of this equation.

In this PhD thesis, we aim at considering more complex versions of the NLS equation, in order to take into account additional physical effects. In particular, we wish to include nonzero boundary conditions at infinity as well as general nonlinearities, such as nonlocal or quasilinear terms. In this context, solitons are named "dark solitons", while the name "bright solitons" is used for the classical solitons (with vanishing boundary conditions at infinity). The main objectives of this PhD proposal are :

- **Numerical computations of dark solitons,** in dimension 2, when the boundary conditions are periodic in one direction and nonzero in the order direction. In the context of [5], we wish to introduce a discretized version of the variational formulation to be able to quantify numerically the transition between "almost 1d" solitons, when the size of the torus is "small", and "actual 2d" solitons, when the size of the torus is "large".
- **Numerical simulation of the dynamics of solitons,** in dimension 2, with nonzero boundary conditions, so as to be able to simulate the dynamics of dark solitons. Some previous work [2,3], in dimensions 1 and 2, exist, that we wish to extend to the theoretical context of [5]. Then, the goal will be to derive, analyze and implement numerical methods in dimension 2 allowing to simulate interactions between solitons (either between dark solitons or between dark and bright solitons), in order to pave the way for a future theoretical analysis.
- **Introduction of more complex nonlinear terms,** in the NLS equation. In particular, we want to be able to consider numerically nonlocal nonlinear terms, as well as quasilinear terms. Examples of nonlocal nonlinear terms can be found in [4] (1 dimensional case), and examples of quasilinear terms can be found in [6] (1 dimensional case as well). With theses additional terms, once their numerical discretization is analyzed, we wish to both compute numercal solitons (axis 1 above) and simulate the dynamics of the equation (axis 2 above) when the boundary conditions at infinity are nonzero, in dimension 2.

Travel expenses are covered within the limits of the scale in force.

Assignment

Assignments :
With the help of G. Dujardin and A. De Laire, the recruited person will be taken to get familiar with the scientific literature on the discretization of NLS equations, to develop, analyze and implement (Python, C, etc) numerical methods, to write scientific articles, to present their work in scientific conferences.

For a better knowledge of the proposed research subject :
A minimal biograhy consists in :

[1] M. J. Ablowitz. Nonlinear Dispersive Waves: Asymptotic Analysis and Solitons. Cambridge Texts in Applied Mathematics. Cambridge University Press, 2011.

[2] W. Bao. Numerical methods for the nonlinear Schrödinger equation with nonzero farfield conditions. Methods and Applications of Analysis, 11(3):367–388, 2004.

[3] W. Bao, Q. Tang, and Z. Xu. Numerical methods and comparison for computing dark and bright solitons in the nonlinear schrödinger equation. Journal of Computational Physics, 235:423–445, 2013.

[4] A. de Laire, G. Dujardin, and S. López-Martínez.Numerical computation of dark solitons of a nonlocal nonlinear Schrödinger equation. To appear in Journal of Nonlinear Science, 2023.

[5] A. de Laire, Philippe Gravejat, and D. Smets. Minimizing travelling waves for the Gross-Pitaevskii equation on R × T. To appear in Annales de la Faculté des Sciences de Toulouse.

[6] A. de Laire and E. Le Quiniou. Exotic traveling waves for a quasilinear Schrödinger equation with nonzero background. Preprint arXiv:2311.08918

Collaboration :
The recruited person will be in connection with G. Dujardin and A. De Laire who will supervize the PhD thesis.

Responsibilities :
The person recruited is responsible for achieving this scientific project.

Skills

Technical skills and level required : M2

Languages : French, English, or Spanish

Relational skills : listening, adaptation, reactivity, autonomy

Other valued appreciated : scientific curiosity, scienfitic ethics

Benefits package

- Subsidized meals
- Partial reimbursement of public transport costs
- Leave: 7 weeks of annual leave + 10 extra days off due to RTT (statutory reduction in working hours) + possibility of exceptional leave (sick children, moving home, etc.)
- Possibility of teleworking and flexible organization of working hours
- Professional equipment available (videoconferencing, loan of computer equipment, etc.)
- Social, cultural and sports events and activities
- Access to vocational training
- Social security coverage

Remuneration

2100€ gross per month for the 1st and 2nd years
2190€ gross per month for the 3rd year

General Information

- **Theme/Domain :** Numerical schemes and simulations
  Scientific computing (BAP E)
- **Town/city :** Villeneuve d'Ascq
- **Inria Center :** Centre Inria de l'Université de Lille
- **Starting date :** 2024-10-01
- **Duration of contract :** 3 years
- **Deadline to apply :** 2024-04-30

Contacts

- **Inria Team :** PARADYSE
- **PhD Supervisor :**
  Dujardin Guillaume / Guillaume.Dujardin@inria.fr

About Inria

Inria is the French national research institute dedicated to digital science and technology. It employs 2,600 people. Its 200 agile project teams, generally run jointly with academic partners, include more than 3,500 scientists and engineers working to meet the challenges of digital technology, often at the

interface with other disciplines. The Institute also employs numerous talents in over forty different professions. 900 research support staff contribute to the preparation and development of scientific and entrepreneurial projects that have a worldwide impact.

The keys to success

There future PhD candidate will have a taste for applied mathematiecs, modelling, numerical analysis and/or scientific computing. This will be clear from his previous scientific formation.

Warning : you must enter your e-mail address in order to save your application to Inria. Applications must be submitted online on the Inria website. Processing of applications sent from other channels is not guaranteed.

Instruction to apply

CV + cover letter

Defence Security :
This position is likely to be situated in a restricted area (ZRR), as defined in Decree No. 2011-1425 relating to the protection of national scientific and technical potential (PPST).Authorisation to enter an area is granted by the director of the unit, following a favourable Ministerial decision, as defined in the decree of 3 July 2012 relating to the PPST. An unfavourable Ministerial decision in respect of a position situated in a ZRR would result in the cancellation of the appointment.

Recruitment Policy :
As part of its diversity policy, all Inria positions are accessible to people with disabilities.