



**Offer #2025-09046**

## **PhD Position F/M Large-time behavior and large-population limits of non-exchangeable particle systems**

**Contract type :** Fixed-term contract

**Level of qualifications required :** Graduate degree or equivalent

**Fonction :** PhD Position

### **Context**

The PhD project will be carried out in the project-team MUSCLEES at the Centre Inria de Paris under the joint supervision of Nastassia Pouradier Duteil (CR Inria) and Benoît Bonnet-Weil (CR CNRS) within the framework of the ANR project FISH (self-organization of Interacting particle Systems with Heterogeneity).

### **Assignment**

Many living systems exhibit fascinating dynamics of collective behavior during locomotion, from bacterial colonies to human crowds. The celebrated Cucker-Smale model describes the dynamics of a group of  $N$  interacting particles whose positions and velocities satisfy a system of differential equations representing the equations of motion.

The particles are said to be *exchangeable* (or identical) if they can be relabeled without impacting the global dynamics. In the exchangeable case, the Cucker-Smale system is known to exhibit a flocking behaviour, that is the asymptotic alignment of all the individual agent velocities, under a “fat-tail” condition on the interaction kernel, see for instance the surveys [4, 5, 6]. These results were extended to the non-exchangeable case in several works including e.g. [1], under some additional conditions on the communication weights.

When the number of interacting agents tends to infinity, the microscopic system can be shown to converge to a continuum limit, which can be written as an integro-differential equation, in which the variables act as labels keeping track of the identities of the individual particles. In this infinite-dimensional framework, the communication weights are replaced by graphons, which can be heuristically understood as generalised adjacency matrices whose evaluation corresponds to the propensity that one agent has to follow another agent.

Using another set of techniques (known as the non-exchangeable mean-field limit), the same microscopic system can be shown to converge to the solution to a non-local transport-type partial differential equation.

- [1] B. Bonnet and É. Flayac. Consensus and Flocking under Communication Failures for a Class of Cucker-Smale Systems. *System and Control Letters*, 152:104930, 10, 2021.
- [2] B. Bonnet, N. Pouradier Duteil, and M. Sigalotti. Consensus Formation in First-Order Graphon Models with Time-Varying Topologies. *Mathematical Methods and Models in Applied Sciences*, 32(11):2121–2188, 2022.
- [3] L. Boudin, F. Salvarani, and E. Trélat. Exponential Convergence Towards Consensus for Non-Symmetric Linear First-Order Systems in Finite and Infinite Dimensions. *SIAM Journal on Mathematical Analysis*, 54(3):2727–2752, 2022.
- [4] Y.-P. Choi, S.-Y. Ha, and Z. Li. Emergent Dynamics of the Cucker-Smale Flocking Model and its Variants. *Active Particles, Volume 1: Advances in Theory, Models, and Applications*, pages 299–331, 2017.
- [5] S.-Y. Ha, K. Lee, and D. Levy. Emergence of Time-Asymptotic Flocking in a Stochastic Cucker-Smale System. *Comm. Math. Sci.*, 7(2):453–469, 2009.
- [6] S. Motsch and E. Tadmor. Heterophilous Dynamics Enhances Consensus. *SIAM Review*, 56(4):577–621, 2014.

## Main activities

The first goal of this PhD is to extend the existing results of convergence to flocking for the microscopic system to its continuum limit. Following the insights garnered in [2], a first natural lead to explore will be that of time-independent coefficients with positive scrambling, which correspond to topologies in which every pair of agents follows a common third party individual. Another relevant setting to investigate is that of interaction topologies with positive Fiedler number, following [1], wherein the sufficient well-connectedness of the system is understood in terms of connectivity properties of the underlying graph, see also [3] for a graphon counterpart of this object.

Further objectives will include (depending on the student's appetite):

- Studying convergence to flocking in the framework of the mean-field limit equation;
- Studying other types of collective behavior introduced by the system's non-exchangeability
- Studying the control of the particle system in order to drive the group to a predetermined collective configuration;
- Deriving rigorously the mean-field limit of more general non-exchangeable particle systems.

## Skills

The applicant should have a solid background in the analysis of Ordinary Differential Equations (ODEs) and Partial Differential Equations (PDEs). Depending on the evolution of the PhD, a more general interest for Measure Theory and Functional Analysis, the study of infinite dimensional systems through the lens of Control Theory, or a keen interest in the practical modelling of Collective Dynamics would be very welcomed traits.

## Benefits package

- Subsidized meals
- Partial reimbursement of public transport costs
- Leave: 7 weeks of annual leave + 10 extra days off due to RTT (statutory reduction in working hours) + possibility of exceptional leave (sick children, moving home, etc.)
- Possibility of teleworking and flexible organization of working hours
- Professional equipment available (videoconferencing, loan of computer equipment, etc.)
- Social, cultural and sports events and activities

## General Information

- **Theme/Domain** : Modeling and Control for Life Sciences  
Biologie et santé, Sciences de la vie et de la terre (BAP A)
- **Town/city** : Paris
- **Inria Center** : [Centre Inria de Paris](#)
- **Starting date** : 2025-09-01
- **Duration of contract** : 3 years
- **Deadline to apply** : 2025-07-22

## Contacts

- **Inria Team** : [MUSCLEES](#)
- **PhD Supervisor** :  
Pouradier Duteil Nastassia / [nastassia.pouradierduteil@inria.fr](mailto:nastassia.pouradierduteil@inria.fr)

## About Inria

Inria is the French national research institute dedicated to digital science and technology. It employs 2,600 people. Its 200 agile project teams, generally run jointly with academic partners, include more than 3,500 scientists and engineers working to meet the challenges of digital technology, often at the interface with other disciplines. The Institute also employs numerous talents in over forty different professions. 900 research support staff contribute to the preparation and development of scientific and entrepreneurial projects that have a worldwide impact.

**Warning** : you must enter your e-mail address in order to save your application to Inria. Applications must be submitted online on the Inria website. Processing of applications sent from other channels is not guaranteed.

## **Instruction to apply**

### **Defence Security :**

This position is likely to be situated in a restricted area (ZRR), as defined in Decree No. 2011-1425 relating to the protection of national scientific and technical potential (PPST). Authorisation to enter an area is granted by the director of the unit, following a favourable Ministerial decision, as defined in the decree of 3 July 2012 relating to the PPST. An unfavourable Ministerial decision in respect of a position situated in a ZRR would result in the cancellation of the appointment.

### **Recruitment Policy :**

As part of its diversity policy, all Inria positions are accessible to people with disabilities.